# Probing the FFLO phase by double occupancy modulation spectroscopy

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We propose here that for a spin-imbalanced two-component attractive Fermi gas loaded in a 1D optical lattice in presence of an harmonic confining potential, the observation of the change in the double occupancy after a lattice depth modulation can provide clear evidence of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase. Simulating the time evolution of the system, we can characterize the double occupancy spectrum for different initial conditions, relating its features to the FFLO wavevector $q$. In particular, the narrowing of the width of the spectrum can be related, through Bethe-ansatz equations in the strongly interacting limit, to the FFLO wavevector $q$.

Ultracold atoms trapped in optical lattices have become an important tool to mimic strongly correlated condensed matter systems, leading to the possibility to explore regimes unattainable within the traditional solid state framework. Recently, a considerable experimental effort [1, 2] has been devoted to the analysis of two-component spin-imbalanced Fermi gases. Theoretical investigations [3–12] have revealed that, in the characterization of 1D spin-imbalanced Fermi gases, a major role is played by the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [13]. In a solid-state context the FFLO phase has been investigated in heavy Fermions systems [14], with techniques ranging from heat capacity to nuclear magnetic resonance measurements, even though conclusive evidence of its existence is still missing. In ultracold gases, even though it has been suggested that its presence can be detected through various measurements such as noise correlation [7, 9, 15], radio-frequency spectroscopy [10], collective modes analysis [12] , and local density profile measurement [11], no direct experimental evidence of the FFLO phase has been found in these systems. Nevertheless in the experiment conducted at Rice University [2], the density profile of each component has been measured, exhibiting a behavior compatible with the theoretical analyzes focusing on the characterization of the FFLO phase. Compared to previous theoretical suggestions, our approach relies on a simple experimental setup and at the same time provides unequivocal signature of the FFLO phase.

In particular, we propose that a clear experimental evidence of the FFLO phase in a 1D optical lattice can be provided by the measurement of the double occupancy (d.o.),– *i.e.* the number of sites populated by two atoms– after a periodic lattice modulation of the initial state at different frequencies (d.o. modulation spectrum, as proposed in [16]). This technique has been employed to observe the appearance of the Mott gap in a repulsive two-component Fermi gas [17] and it has been suggested as a possible tool to detect the antiferromagnetic phase in such systems [18, 19]. Performing the same kind of experiment for an attractive gas is well within reach of the current experimental techniques and, as we will show here, can provide clear evidence of the FFLO phase through a reduction of the width in the d.o. spectrum directly related to the FFLO vector $q$. The underlying physics is simple: the existence of a collective momentum $q$ restricts the available momentum states of the excitations, thus narrowing the spectrum.

In presence of a parabolic confining potential, we assume that the system is described by the Hubbard Hamiltonian

$$H = H_J + H_U + \sum_i^L V_i \left( n_{i\uparrow} + n_{i\downarrow} \right), \qquad (1)$$

where $H_J = -J \sum_{i,\sigma=\uparrow\downarrow}^L c_{i\sigma}^\dagger c_{i+1\sigma} + h.c.$, $H_U = -U \sum_i^L n_{i\uparrow} n_{i\downarrow}$, $V_i = V(i - \frac{L}{2})^2$, $J$ is the hopping amplitude, $-U$ is the on-site attractive interaction, $V$ the global confining potential, and the polarization $P$, defined as $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$, with $N_\sigma = \sum_\sigma n_{i\sigma}$.

The lattice depth modulation proposed in [16, 17] can be modeled by the modulation of the hopping amplitude $J(t) = J + \delta J \cos(\omega t)$ [18, 19]. Since we are interested in excitations which lead to pair breaking, it is natural to focus on modulation frequencies close to the energy $U$, related to pairing, *i.e.* we concentrate on the transition between the first and the second Hubbard band. Intuitively, the process we are interested in might be understood as the transition between the ground state and a state where a pair has been broken by the hopping modulation.

Our approach to the problem is twofold. We first perform numerical simulations of the ground state and of the dynamical evolution of the system. We then move to the analysis of the results in terms of Bethe-ansatz (BA), in the limit $U/J \to \infty$. The numerical simulations, both for the ground-state calculation and for the time evolution, are performed with the aid of a time-evolving block decimation (TEBD) code [20], which can be regarded as a quasi-exact method for the analysis of 1D quantum systems. In the spin-polarized case, for the range of parameters that we have considered ($U = -10$, $J = 1$, $N_\uparrow + N_\downarrow = 40$, $P \geq 0.04$, $V = 0.005$) the ground state consists of a central region where $\langle n_{i\uparrow} \rangle > \langle n_{i\downarrow} \rangle > 0$ and an outer,
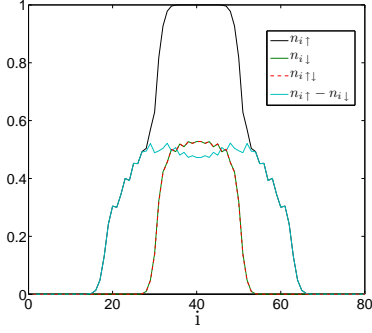
FIG. 1: (color online) Particle and pair densities and the difference $n_{i\uparrow} - n_{i\downarrow}$ for the polarization $P = 0.5$ ($N_\uparrow = 30, N_\uparrow = 10$).
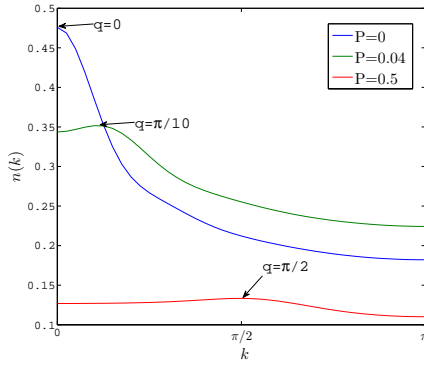


FIG. 2: (color online) Pair momentum distribution $n(k)$ for the polarizations $P = 0$, $P = 0.04$, $P = 0.5$. The maxima are at $q = 0, \simeq \frac{\pi}{10}, \simeq \frac{\pi}{2}$, respectively.

fully polarized region ($\langle n_{i\uparrow} \rangle > 0$, $\langle n_{i\downarrow} \rangle = 0$). Moreover, in the central region of the trap $\langle n_{i\downarrow} \rangle \simeq \langle n_{i\uparrow} n_{i\downarrow} \rangle$ implying that due to the strong interaction considered here all minority particles $n_{i\downarrow}$ are paired (Fig. 1). The periodic spatial dependence of $n_{i\uparrow} - n_{i\downarrow}$ suggests the presence of the FFLO state [21]. In order to give a quantitative estimate, we extract the value of the FFLO wavevector $q$ from the pair correlation function $\langle c_{j\uparrow}^\dagger c_{j\downarrow}^\dagger c_{i\uparrow} c_{i\downarrow} \rangle$ and its Fourier transform $n_{pair}(k)$, defining $q$ as the maximum in the distribution $n_{pair}(k)$ (Fig. 2).

We first examine the properties of the system for $P = 0$. After calculating the ground state for the Hamiltonian, we turn on the modulation of the hopping amplitude. At each timestep we calculate the total d.o. spectrum $D_\omega(t) = \sum_{i=1}^L \langle n_{i\uparrow} n_{i\downarrow} \rangle$, where the modulation frequency $\omega$ is centered around the value of the interaction strength $|U|$. In Fig. 3, the d.o. spectrum $\overline{D}_\omega(t) \mid_{t=50}$ is plotted for frequencies $\omega \in [0.5; 1.8]$, where $\overline{D}_\omega(t) \mid_{t=50}$ is the average between the local maxima and minima in the small-time dynamics of the d.o.. The spectrum shows a band-like structure with $\omega_{min} \simeq 0.68$ and $\omega_{max} \simeq 1.5$. As we will later show, the band in Fig. 3 can be explained
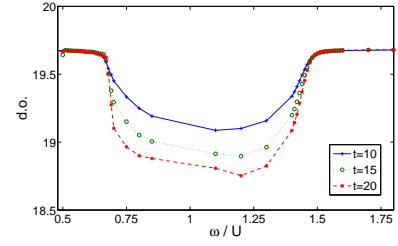


FIG. 3: (color online) Double occupancy $\overline{D}_\omega(t)$ as function of frequency $\omega$ for times $t = 10, 15, 20$ for the balanced case $N_\uparrow = N_\downarrow = 20$.

in terms of the excited states within the second Hubbard band.

We will now turn our attention to the numerical results for the double occupancy spectrum of the spin-polarized case (Fig.4). The first important aspect is the decrease in the reduction of the d.o. as the number of paired particles is increased (see Fig. 4). This feature can be easily understood considering that the lattice modulation at frequencies close to $U$ affects the paired component of the gas only and hence the number of broken pairs is reduced accordingly. However, the most prominent feature of the spectrum in the spin polarized case is the reduction of the width of the band. In particular, while the position of its upper limit is independent of the polarization, the lower limit depends strongly on $P$. The main goal of our analysis is to show that the width of the band $\Delta\omega$ can be described by the relation $\frac{\Delta\omega}{U} = \frac{4J}{U}(1 + \cos q)$, where $q$ is the FFLO wavevector, calculated from the ground-state value of $n_{pair}(k)$. We thus claim that the determination of the d.o. modulation spectrum in an imbalanced gas allows the direct determination of the $q$ vector characteristic of the FFLO phase.

The physical situation depicted here can be analyzed in terms of the mapping between the attractive and the repulsive Hubbard model. Changing $U \to -U$, the single-site basis states can be mapped according to the following scheme

$$| \uparrow\downarrow \rangle \leftrightarrow | \uparrow \rangle, \quad |\emptyset\rangle \leftrightarrow | \downarrow \rangle. \qquad (2)$$

For repulsive interaction, the hopping modulation results in an increase of the d.o., since, in that case, the modulation cause the formation of a doubly occupied and an empty site [18, 19]. In the case analyzed here the opposite process takes place: a doubly occupied/empty site "pair" is broken. However, as a consequence of the mapping, the bandwidth for the two processes is the same.

In order to explain the results obtained we will consider here the BA solution for the open-boundary conditions (OBC) Fermi-Hubbard model in the limit $U/J \to \infty$. In the case $U > 0$, it is possible to prove that the excitations of the system can be described in terms of $N = N_\uparrow + N_\downarrow$ spinless fermions with energy and momenta given respec-
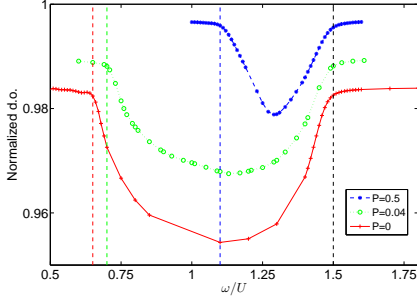
FIG. 4: (color online) Normalized double occupancy $\overline{D}_\omega(t)/N_\downarrow$ as a function of frequency $\omega$ at $t = 10$ in a trap for three cases with polarizations $P = 0$, 0.04, 0.5 corresponding to $N_\downarrow = 20$, 22, 10. The vertical lines correspond to $E_{high}$ (black line), and to $E_{low}$ for different polarizations. As mentioned in the text, the (normalized) local minimum for $\overline{D}_\omega(t)$ decreases for increasing number of pairs. Specifically it is located at 19.66, 21.76, 9.96 for $N_\downarrow = 20$, 22, 10 respectively.

tively by

$$E = -2J \sum_{j=1}^{N} \cos k_j,$$

$$k_j = \frac{\pi}{L+1} I_j \quad I_j \in \mathbb{N}, j = [1 \ldots N], \quad (3)$$

where Eq. (3) can be directly obtained from the $U/J \to \infty$ limit of the BA equations (see Supplementary Information). The distribution of $I_j$ should correspond to a condition where energy is minimized. In the half-filled case, the energy minimization condition is given by $I_j = [1 \ldots L]$, leading to $E = -2J \sum_{j=1}^{L} \cos k_j = 0$, $p = \sum_{j=1}^{L} k_j$.

We now turn to the analysis of the attractive interaction case. Following the mapping described in Eq. (2), the total number of up spins $N_\uparrow$ in the repulsive case maps to the total number of pairs $N_{\uparrow\downarrow}$ and $N_\downarrow$ to the number of empty sites $N_\emptyset$, leading to $N = N_\emptyset + N_{\uparrow\downarrow}$. In the strongly attractive regime, we can assume that all down particles are paired, leading to $N_{\uparrow\downarrow} = N_\downarrow$. $N_\emptyset$ is the number of sites which are neither occupied by a pair ($N_{\uparrow\downarrow} = N_\downarrow$) or by an unpaired majority atom ($N_\uparrow - N_{\uparrow\downarrow}$), and hence $N = L - (N_\uparrow - N_\downarrow)$, leading to

$$E = -2J \sum_{j=1}^{N_\downarrow - N_\uparrow} \cos k_j, \quad k = \sum_{j=1}^{N_\downarrow - N_\uparrow} k_j. \quad (4)$$

From Eq. (4), it is possible to relate the Fermi momentum for the spinless Fermion gas to the polarization $P$, namely consider $k_F = \pi(N_\uparrow - N_\downarrow)/(L+1)$ and then observe that the FFLO momentum, defined as $q = \pi\rho P$ with $\rho = (N_\uparrow + N_\downarrow)/L$, coincides with $k_F$ (if $L \simeq L+1$). Obviously, for a half-filled system $N_\uparrow + N_\downarrow = L$ and hence
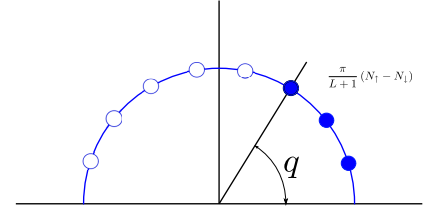


FIG. 5: Schematic representation of the Fermi sea for the spinless Fermions, the Fermi wavevector lies at $q$

$k_F \simeq \pi P = q$. Even the situation where a parabolic confining potential is present can be intuitively understood in terms of an "effective" FFLO vector, determined by the spatially dependent density of the system (see [8]).

The effect of the hopping modulation is to create two *fermionic* excitations (corresponding to the up and down fermions originating from the breaking of the pair) above the Fermi energy of the spinless Fermions given in Eq. (4). The change of the kinetic energy imposed by the presence of these two excitations with respect to the ground state is given by

$$\Delta E_{kin} = -2J(\cos k_1 + \cos k_2) \quad (5)$$

with $-1 \leq \cos k_{1,2} < \cos q$. In addition to $\Delta E_{kin}$, the pair breaking also involves a change in the interaction energy $\Delta E_{int} = U$. The total energy difference associated to the breaking of the pair can be thus expressed as $\Delta E = -2J(\cos k_1 + \cos k_2) + U$. We then expect that in a d.o. modulation experiment for an imbalanced gas the pair breaking band will lie between $E_{low} = U - 4J \cos q$ ($k_1 = q$, $k_2 = q$) and $E_{high} = U + 4J$ ($k_1 = \pi$, $k_2 = \pi$), leading to a bandwidth

$$\frac{\Delta \omega}{U} = \frac{4J}{U}(1 + \cos q). \quad (6)$$

In addition, the finite value of $U/J$ in the numerical simulations implies a shift in the d.o. spectrum $U \to U^*$ leading to

$$E_{low} = U^* - 4J \cos q, \quad E_{high} = U^* + 4J, \quad (7)$$

keeping the value of $\Delta \omega$ unchanged. This shift is connected to the shift of the ground-state energy within the first Hubbard band induced by the finite value of the ratio $U/J$. The explicit calculation of the Hubbard spectrum for a two-site system allows to get a qualitative understanding of the physical reason behind this phenomenon. More specifically, the finite value of $U/J$ implies a lowering of the ground-state energy with respect to the case $U/J \to \infty$, along with a removal of the degeneracy connected to the spin degree of freedom (see Supplementary Information). The relation given by Eq.(6) as well as the values $E_{low}$ and $E_{high}$ are nevertheless still valid. In Fig. 4 it is possible to observe how the numerical results correspond to our analytical description.
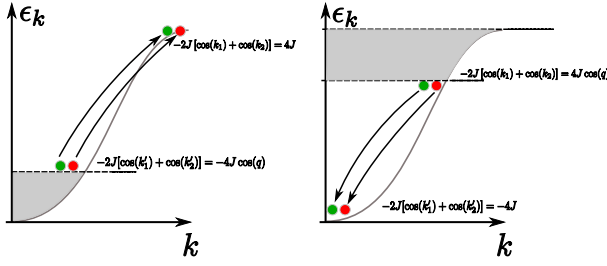
FIG. 6: (left) Representation of the scattering process between two particles with initial momentum $k_1 = q$ and $k_2 = q$ and final momentum $k_1' = \pi$ and $k_2' = \pi$, corresponding to the kinetic energy transfer $\Delta E_{max} = 4J + 4J \cos q$ . The pair momentum $q$ restricts the available initial states. (right) Scattering process between the states $k_1 = \pi + q$, $k_2 = -\pi + q$ and $k_1' = 0$ and $k_2' = 0$.

Intuitive understanding of the BA results can be provided by considering a related example, namely the inelastic scattering of particles (Fig. 6). A restriction imposed on the possible values of the momenta –in our case dictated by the FFLO wavevector $q$ of the initial state– implies a reduction of the bandwidth associated with the scattering process. If the particles considered have initial momenta and final momenta $k_1$, $k_2$ and $k_1'$, $k_2'$ respectively, the maximum kinetic energy change in the scattering process will be (assuming the lattice dispersion relation) $\Delta E_{max} = 4J + 4J \cos q$ (for $k_1 = q$, $k_2 = q$, $k_1' = \pi$, $k_2' = \pi$), and the minimum will be given by $\Delta E_{min} = -4J - 4J \cos(q)$ (for $k_1 = \pi + q$, $k_2 = -\pi + q$, $k_1' = 0$, $k_2' = 0$), hence the largest possible difference in the kinetic energy associated to the scattering process will be given by $\Delta E_{max} - \Delta E_{min} = 8J(1 + \cos q)$. This simple example illustrates how, in general, the limitation of the accessible momentum states is reflected in the reduction of the bandwidth.

Since we are addressing a possible experimental setup to detect the FFLO phase in 1D, it is necessary to address the role of temperature. In [9, 22], the temperature stability of the FFLO in 1D traps has been considered. In particular in [9] the transition temperature $T_c$ between a phase-separated FFLO+normal $\rightarrow$ normal phase is discussed, leading to $T_c \simeq 0.2 T_F$. This result is obtained within a mean-field picture, providing an approximate upper limit of the temperatures needed to observe the FFLO phase. This range of temperature seem to be well within reach in present experiments. In [2] temperatures $\simeq 0.1 T_F$ have been reported, suggesting that the FFLO phase could be observed in the near future.

Through a combination of numerical simulations and analytical results expressed in terms of BA equations, we have been able to relate the d.o. modulation spectrum to the presence of a FFLO state, giving a quantitative estimate of the bandwidth narrowing in terms of the wavevector $q$. Our analysis establishes the first simple clear experimental tool to detect and quantita-

tively characterize the FFLO phase in ultracold gases in quasi 1D optical lattices. It also shows, on more general grounds, how a collective (pair) momentum can be related to observable quantities in 1D systems in a simple and clear manner.

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